4-8 Videos Guide

4-8a

- Rectangular-spherical conversions
  - $x = \rho \cos\theta \sin\phi$
  - $y = \rho \sin\theta \sin\phi$
  - $z = \rho \cos\phi$
  - $x^2 + y^2 + z^2 = \rho^2$

4-8b

Exercise:

- Sketch the solid described by the given inequalities.
  $1 \le \rho \le 2, \quad \pi/2 \le \phi \le \pi$

- The triple integral in spherical coordinates
  - $\iiint_E f(x,y,z)\, dV$
    $= \int_c^d \int_\alpha^\beta \int_{g_1(\theta,\phi)}^{g_2(\theta,\phi)} f(\rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi)\, \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$
  - Note that $dV = \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$

Exercises:

4-8c

- Sketch the solid whose volume is given by the integral and evaluate the integral.
  $\int_0^{\pi/4} \int_0^{2\pi} \int_0^{\sec\phi} \rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$

4-8d

- Use spherical coordinates to evaluate $\iiint_E y^2 x^2\, dV$, where E lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 1$.

4-8e

- Evaluate the integral by changing to spherical coordinates.
  $\int_{-a}^{a} \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2 z + y^2 z + z^3)\, dz\, dx\, dy$